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THE EXPECTED LENGTH OF A MYOPIC INVESTIGATION

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

THE EXPECTED LENGTH OF A MYOPIC INVESTIGATION

by

Peter Carlton Olsen

September 1978

Thesis Advisor: F. Russell Richards

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This thesis addresses the problem of estimating the distance which must be traveled to visit all the targets in a search area. The search area is assumed to be a compact area drawn from an infinite Poisson field. Targets are visited in "nearest uninvestigated neighbor" order. A simplified analytical model is developed for the unbounded investigation. This model is used to provide heuristic justification for simulation results obtained for the bounded investigation.

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The Expected Length of a Myopic Investigation

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ABSTRACT

This thesis addresses the problem of estimating the distance which must be traveled to visit all the targets in a search area. The search area is assumed to be a compact area drawn from an infinite Poisson field. Targets are visited in "nearest uninvestigated neighbor" order. A simplified analytical model is developed for the unbounded investigation. This model is used to provide heuristic justification for simulation results obtained for the bounded investigation.

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I. INTRODUCTION

A. GENERAL DESCRIPTION OF THE PROBLEM

This thesis addressed the problem of estimating the distance which must be traveled to visit targets which are randomly distributed over a large area. The problem is interesting because many practical search operations require the investigation of targets after detection. It is often necessary to estimate the time which will be required for the entire operation before the targets are even detected.

Classical search theory addresses cases where it can be assumed that there are few targets in the area, and that all are of equal interest when detected. Later classification may rule out some targets as false targets. This is not usually obvious to the searcher. Any detection will stop the search. Subsequent classification as a false alarm may result in its restart.

A variation on this case occurs when there are many false targets, but they may be classified immediately upon detection. If the target may be immediately classified as false, then the search can proceed unimpeded. False targets can be treated as if they had never been detected. If all false targets are immediately classified as false, then this case becomes equivalent to the classical case of sparse targets. The search continues until the true target is found or the area is exhausted. When the first target is detected which cannot be dismissed as false, the search is stopped and that target is pursued.

There is a third case which shall be addressed. It is intermediate between the first two. It can be defined as follows: targets

can be easily classified, but only at very close range. The searcher can detect targets at long range, but, for classification, must take a "close look" at which time he knows immediately whether or not he has found the object of his search. If he has, he stops, if he hasn't, he keeps searching. If the targets are dense enough, or the searcher's detection range great enough, the searcher may always have uninvestigated targets in sight. He may not really "search" at all in the sense of classical theory; he may instead travel directly from target to target without any regular "search pattern." The problem is now one of "investigation theory" as first defined by Nuendorfer [1]. The investigation will continue until the desired object has been found, or until all of the targets have been visited. In either event, the investigator's track will have gone to the location of each prior false alarm.

This is the crux of the problem. If the positions of the targets are unknown until detection, then the length of the entire path will be uncertain. If the target positions are random variables, then the length of the path required to investigate the targets will also be a random variable.

B. EXAMPLE

The case described above is typical of many Coast Guard operations. The Coast Guard is often required to search for a particular boat. Usually the boat can be positively identified only by name or registration number. Small boats which have been reported overdue, and boats suspected of law enforcement violations are two classes which must be searched for in this manner. There may be many boats which fit a general description, and the searcher may have to check out all of them.

A simple scenario can be constructed which illustrates the problem. Consider a Coast Guard Cutter with embarked helicopter assigned to patrol a ten thousand square mile area for fishing violations. The helicopter must be fueled tonight for tomorrow's first light patrol. The aircraft commander must decide how much fuel to load. Too little fuel means an incomplete patrol. Too much fuel means that something else must be left behind. Experience has shown that rarely are more than twelve or fewer than eight ships seen. Ten is the average number. The aircraft commander has no way of predicting the future position of any of the fishing vessels, but he must still make an important operational decision, based on the length of the path which he expects to fly. The result developed in this paper would be directly applicable in this scenario.

II. ANALYSIS

A. ANALYTICAL FORMULATION OF THE PROBLEM

Two basic questions must be addressed first off:

1. What distribution is to be assumed for the positions of the targets?

2. In what order are the targets to be visited?

In this thesis it is assumed that the area considered is a portion of an infinite Poisson field with targets distributed with spatial Poisson density ρ. This means that the number of targets in any finite area, A, will be a random variable, with a Poisson distribution and mean ρA. P.A.W. Lewis and G.S. Shedler [2] have shown that the targets occurring in an area as the result of this process will be uniformly distributed within the area. This is to say that if the entire area is divided up into many small boxes, each large enough to hold only one target, then each box has an equal chance of actually holding one target. This distributional assumption is attractive because it assumes the minimum amount of information about the target's location. In this sense it is the "worst case"; more information can only improve the situation. In actual Coast Guard practice, there are many searches conducted when uniformity is the best assumption which can be made; the actual distribution of targets, including false targets, cannot be distinguished from what would have been expected from a uniform distribution.

A related question is that of target motion. In this thesis it will be assumed that the targets move so much more slowly than the investigator that they may be considered stationary.

The question of the order in which targets are to be visited is more complicated. If the targets are stationary, there is one unique optimum order resulting in the minimum total path length. The problem of determining this optimum ordering is called the "travelling salesman problem." The travelling salesman solution, while ideal, is not practical for large problems. There is no efficient method of solution. Every current method is fundamentally equivalent to trying all possible orders [3]. There are some shortcuts for small numbers of points; the fact that points on the perimeter will be taken in the relative order that they are found around the perimeter is an example. But these shortcuts do not work for large numbers of points with complex geometry. Determining the optimum path through twenty points will require a tremendous amount of calculation.

This thesis employs the myopic decision rule in which the investigator will always go to the nearest point which has not yet been visited. This decision rule is justified on two grounds:

1. It is easily employed in actual searches; the searcher need only remember what targets have been investigated, and then select the closest of the remainder.

2. Balut [4] has shown simulation results indicating that the path resulting form the myopic decision rule is only about 15% longer than the optimal path for investigations where the investigator is much faster than the targets. This procedure is often used in actual searches.

Finally, the assumption is made that the investigator's detection range is much greater than the inter-target spacing. This means that the investigator will always know the location of the nearest uninvestigated target.

B. MODEL FOR AN UNBOUNDED MYOPIC INVESTIGATION

Because the area considered is a portion of an infinite Poisson field, it is profitable to first consider the properties of an investigation conducted in an infinite field. The investigator will start at one of the points in the field and then visit its closest neighbor, then the closest neighbor remaining, and so on. The investigator will always travel to the nearest uninvestigated target (n.u.t.). Real searches never detect an infinite number of targets, but this is not a bad approximation to the case when the area is very large and there are many more targets detected than can be investigated in the course of a mission.

Consider a Poisson field with spatial density p. Call the set of points $I = \{p_0, p_1, p_2, p_3, \ldots, p_n\}$ an investigation starting at p_0. If $p_n = p_0$, i.e. the investigator returns to the point where he started, then the investigation is a tour. The investigation is conducted according to the following rule: the investigator will always visit the closest point to which he has not yet been. Let r_i be the distance from p_{i-1} to p_i. Let $\ell_k = \sum_{i=1}^{k} r_i$. Let $t_n = \sum_{i=1}^{n} r_i$, when $p_n = p_0$. ℓ_k is the cumulative distance convered in visiting the first k points; t_n is the length of a tour visiting all the points and returning to its starting point.

The expected length of the first radius, r_1, may be calculated as follows. The probability of a target being found in a small area, dA, is

$$P(dA) = pdA.$$

Thus the probability that a target will not be found is

$$\overline{P}(dA) = 1 - pdA.$$

Let $\overline{F}(A)$ be the probability that there is no point in some large area A. The probability that there is no point in (A + dA) is

$$\overline{F}(A) \cdot \overline{P}(dA) = \overline{F}(A)(1 - pdA).$$

But this is also $\overline{F}(A + dA)$

$$\therefore \overline{F}(A)(1 - pdA) = \overline{F}(A + dA)$$

thus

$$\frac{\overline{F}(A + dA) - \overline{F}(A)}{dA} = -p\overline{F}(A).$$

As $dA \to 0$, then this becomes

$$\frac{d\overline{F}(A)}{dA} = -p\overline{F}(A)$$

so that

$$\overline{F}(A) = ce^{-pA}$$

where c is the arbitrary constant of integration.

As $\overline{F}(0) = 1$, then $c = 1$ and

$$\overline{F}(A) = e^{-pA}.$$

The area considered so far has been perfectly general, of arbitrary shape.

Now consider A to be the area circumscribed by a circle of radius r, $A = \pi r^2$. Let A_1 be the smallest such circle tangent to an adjacent point. Let r_1 be its radius. The tail distribution of A_1, $\overline{F}_{A_1}(t)$ is defined as follows:

$$\overline{F}_{A_1}(t) = P(A_1 > t)$$

Let $t = \pi x^2$, where it is the area circumscribed by a circle of radius x.

Then

$$\begin{aligned}\overline{F}_{A_1}(t) &= P(A_1 > t)\\ &= P(\pi r_1{}^2 > \pi x^2)\\ &= P(r_1{}^2 > x^2)\\ &= P(r_1 > x) \quad \text{as} \quad x, r_1 \geq 0.\end{aligned}$$

Define

$$\begin{aligned}\overline{G}_{r_1}(x) &= P(r_1 > x)\\ &= \overline{F}_{A_1}(t).\end{aligned}$$

But

$$\begin{aligned}\overline{F}_{A_1}(t) &= e^{-pt}\\ &= e^{-p\pi x^2}\end{aligned}$$

$\therefore$

$$\overline{G}_{r_1}(x) = e^{-p\pi x^2}.$$

The expected value of r_1, $E(r_1)$, is defined as

$$E(r_1) = \int_0^\infty \left[\frac{-dG_{r_1}(u)}{du}\right] u \, du$$

Integrating by parts yields

$$\begin{aligned}E(r_1) &= \int_0^\infty G_{r_1}(u)du = \int_0^\infty \exp\,[-p\pi u^2]\,du\\ &= \frac{1}{2\sqrt{P}}\end{aligned}$$

where $\overline{G}_{r_1}(x) = P(r_1 > x)$

p = Spatial Poisson density.

Thus the expected radius of the smallest circle containing an adjacent point is

$$E(r_1) = \frac{1}{2\sqrt{-p}}$$

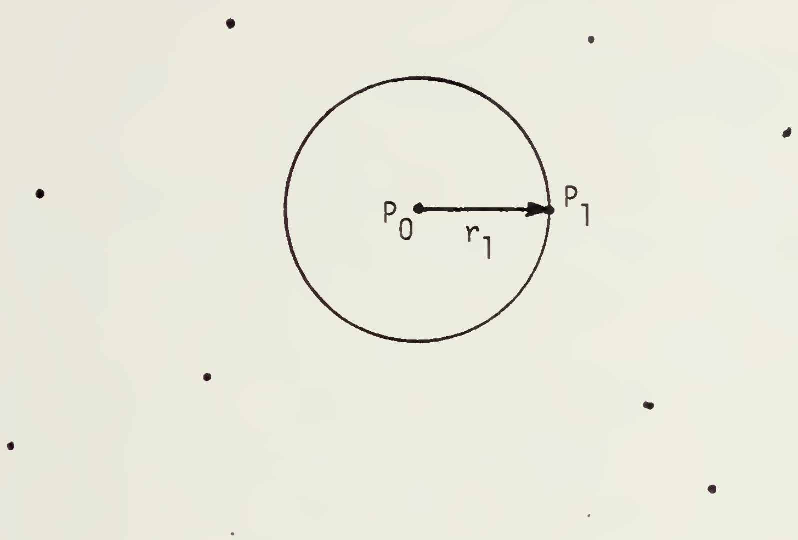


Figure II-1

The expected values of successive jumps are much more difficult to determine. The distribution in the fresh (unsearched) area is still

$$\overline{F}(A) = e^{-pA}$$

but the new area is no longer a circle. Let x_1 be the actual distance between p_0 and its nearest neighbor, p_1. Then a circle of radius x_1, centered at p_0, is known to contain no other points besides p_0. The new point, p_1, lies on to this circle. Area included in this circle "doesn't count" in the search for new points and must be ignored in calculating the distribution function for the next interpoint distance, $\overline{G}_{r_2}(x)$. Let $H_i(x)$ be the function whose value is the new (uninvestigated) area lying within a circle of radius x, centered at P_i.

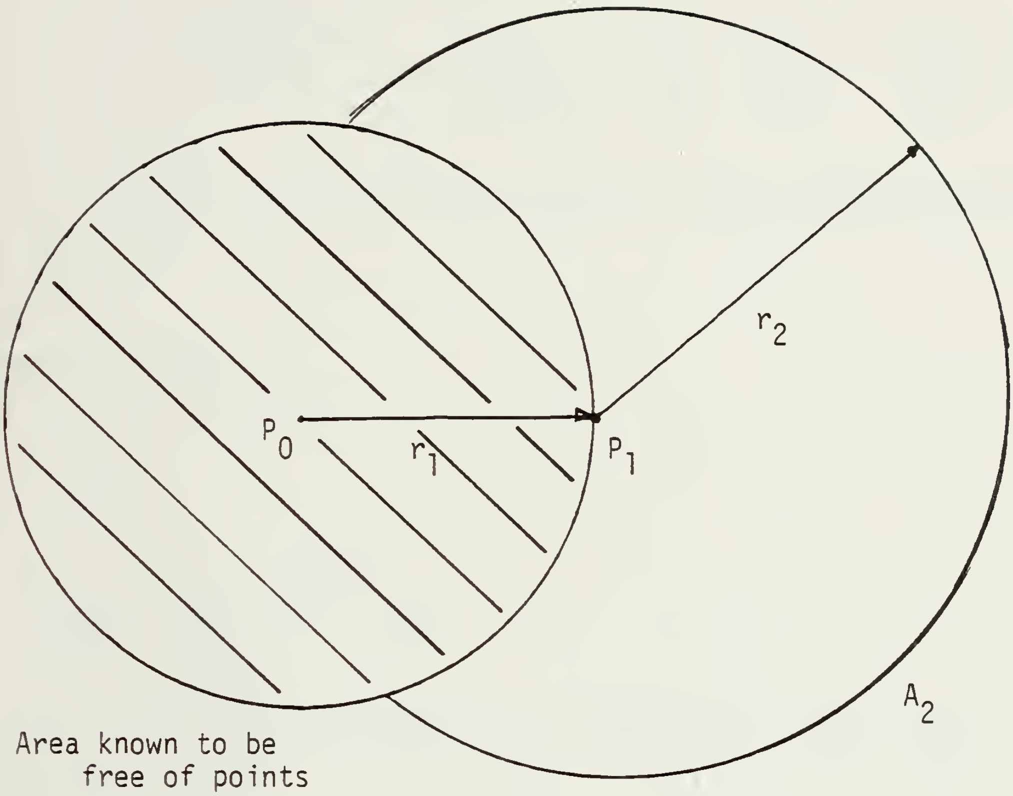


Figure II-2

Here $r_2 < 2r_1$, thus

$$A_2 = H_1(r_2) = \pi(r_2^{\,2} - r_1)^2 - \{2r_1(\frac{r_2^{\,2}}{2r_1^{\,2}} - 1)\cos^{-1}\frac{r_2}{2r_1} - \frac{r_2}{2r_1}\sqrt{1 - \frac{r_2^{\,2}}{4r_1^{\,2}}}\}$$

Thus

$$H_0(x) = \pi x^2 \quad \text{and} \quad \overline{G}_{r_1}(x) = e^{-pH_0(x)}$$

It can easily be seen that

$$H_1(x) = \pi(x^2 - x_1^2) \quad \text{if} \quad (x \geq 2x_1)$$

$$= \pi(x^2 - x_1^2) - \{2x_1(\frac{x^2}{2x_1^2})\cos^{-1}\frac{x}{2x_1} - \frac{x}{2x_1}\sqrt{1 - \frac{x^2}{4x_1^2}}\ \}$$

if $(x < 2x_1)$ where x_1 is the actual observed distance between p_0 and p_1

Let $\overline{G}_{r_2}(x) = P(r_2 > x)$.

Then, as before

$$\overline{G}_{r_2}(x) = e^{-pH_1(x)}$$

and

$$E(r_2) = \int_0^\infty e^{-pH_1(u)}du$$

$$= \int_0^{2x_1} \exp\{-p\{\pi(x - x_1^2) - [2x_1(\frac{x^2}{2x_1^2} - 1)\cos^{-1}\frac{x}{2x_1} - \frac{x}{2x_1}\sqrt{1 - \frac{x^2}{4x_1^2}}\ ]\}\}dx$$

$$+ \int_{2x_1}^\infty \exp\{-p\pi(x^2 - x_1^2)\}dx$$

This expression is difficult to evaluate analytically, and the expected values of r_3, r_4 ... are even more difficult to obtain. This is because the expected value for r_k depends not only upon r_1 and $r_2 \ldots r_{k-1}$, but also upon the angles between successive radii. An investigation which doubled back on itself, Figure II-3(a), would have a different next expected radius than one which did not.

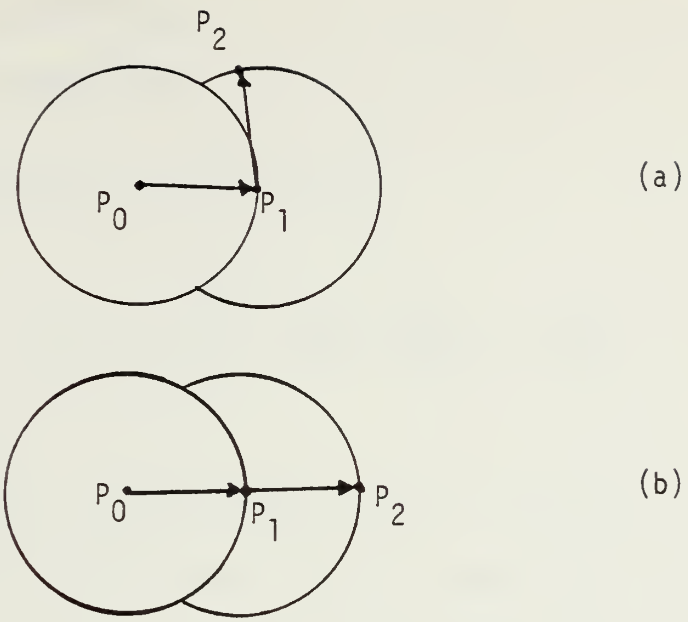


Figure II-3

While the total sanitized area is the same in both cases, it is not distributed according to a common function of radius.

Because the distributions and moments are so difficult, approximations would be useful. An argument can be advanced to support the conjecture that $E(r_i)$, $i = 1, 2, 3, \ldots$ are asymptotically linear as $i \to \infty$. Because new points will be located in an uninvestigated area, the process will tend to creep away from sanitized areas and toward unsanitized areas. This will usually put the investigator at the boundary of a fresh area, with a constantly expanding sanitized area behind him. Asymptotic linearity may be shown easily in a restricted model based on this behavior. Consider a completely deterministic process where points appear on a straight line. The spacing between the points will be determined as follows:

1. r_1 will be arbitrary,

2. r_i will be such that

$$\pi r_1^2 = H_{i-1}(r_i) = \pi(r_i^2 - r_{i-1}^2) -$$

$$\{2r_{i-1}\left(\frac{r_i^2}{2r_{i-1}^2} - 1\right)\cos^{-1}\frac{r_i}{2r_{i-1}} - \frac{r_i}{2r_{i-1}} \sqrt{1 - \frac{r_i^2}{4r_{i-1}}}\}$$

for all in i > 1.

This is equivalent to circumscribing a circle of radius r_1 about the first point, and then adding other points so that the new area added to the process at each step is equal to the initial area enclosed in the first circle. The process can be visualized as a sequence of circles proceeding down a line. Each new circle is centered on the intersection of its predecessor and the centerline. Each circle is sized so that its area, less the area of its predecessors, is equal to r_1^2. Refer to Figure II-4.

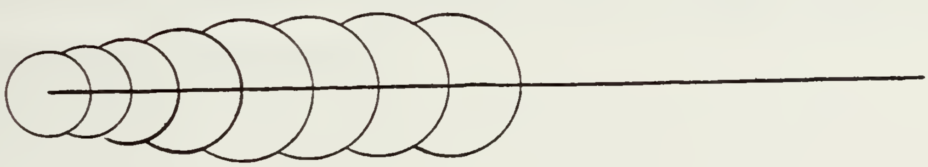


Figure II-4

Numerical calculation for all initial values selected between $r_1 = 1 \times 10^{-1}$ and $r_1 = 1.0 \times 10^{50}$ have shown that the sequence $r_1, r_2, r_3, \ldots, r_n, \ldots$ is asymptotic to $1.2814\ r_1$.

While one would never expect to see an actual realization develop along a perfectly straight line, an extensive computer simulation by Laidlaw [5] does show that many investigations do develop in a way that could be characterized as random deviations on either side of a mean line of progress. It seems reasonable that, if this were so, and the investigation did proceed as variations about a mean line, then the actual limiting value might be $r_n \to cr_1$, where $c \geq 1.2814$. Laidlaw's simulation yielded $c_1 = 1.405$ based on a myopic investigation conducted in a finite plane containing 10,000 points, with the investigation stopping when the boundary of the plane was closer than the nearest target, and with the sweep width much greater than the expected target spacing.

The assumptions for this result were:

1. The points visited are located in an infinite Poisson field.
2. Detection range is <u>much</u> greater than target spacing.
3. Myopic search is employed.

Under these assumptions the length of the total path appears to be asymptotically convergent to

$$\begin{aligned} \ell_n &= ncr_1 \\ &= \frac{n(1.405)}{2\sqrt{p}} \end{aligned}$$

The model is asymptotically linear in n and is called the asymptotic linear model of the unbounded investigation.

C. APPLICATION TO THE BOUNDED INVESTIGATION

The linear asymptotic model cannot be directly applied to the bounded investigation. The linear model requires that the spatial Poisson paramenter, p, remain constant in the unsearched area. This will not be true in a finite area, because the finite area will contain only a finite number of points.

Nevertheless, the linear asymptotic model does suggest an indirect approach. Consider a bounded investigation to consist of two parts:

1. An investigation of the points near the center of the area where the unbounded linear model may be applied.

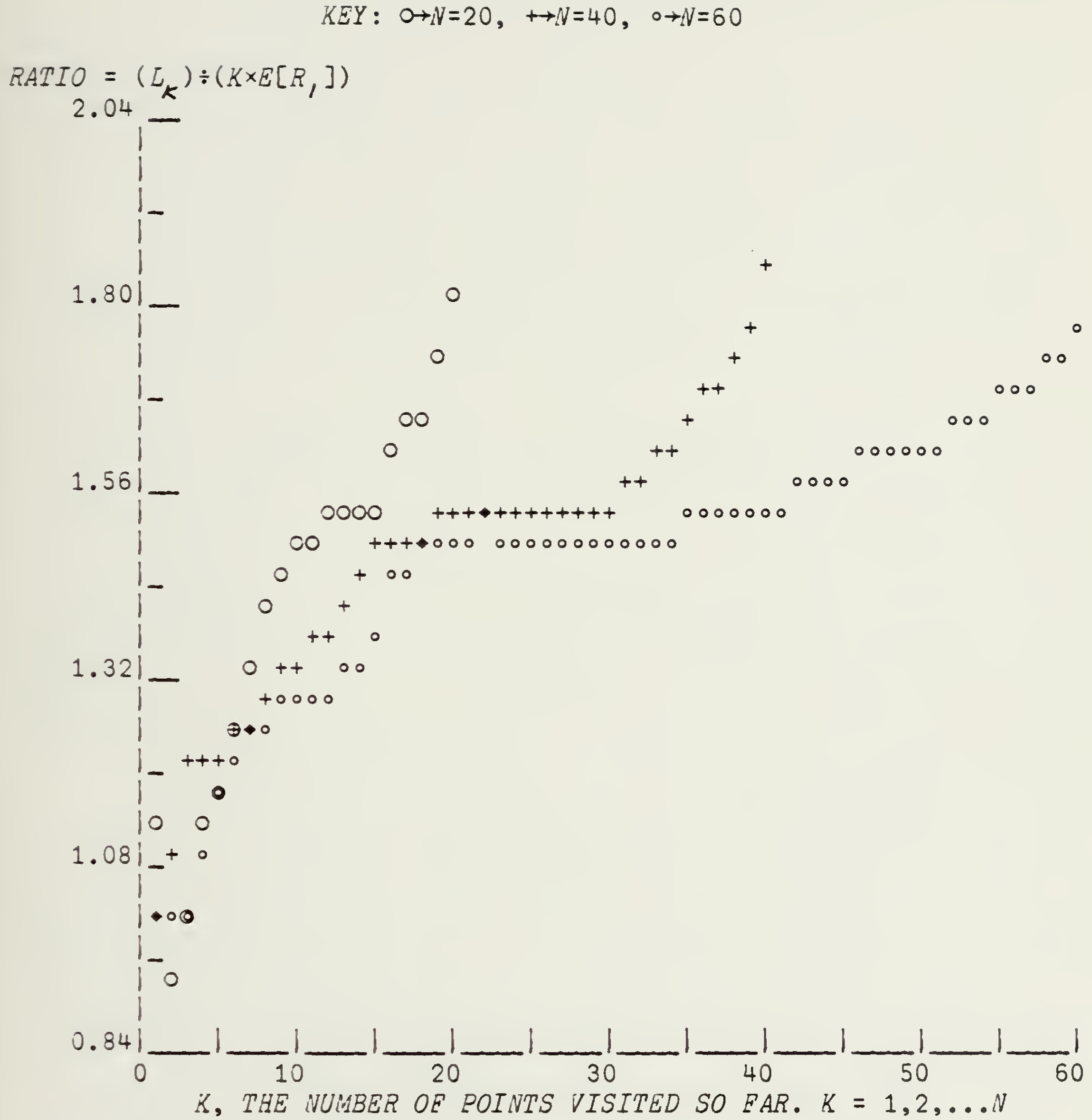
2. An investigation of the remaining points near the boundary.

It seems reasonable to suppose that the bounded investigation might first behave like the unbounded case, when there are a large number of points from which to draw. After a while the points will have been "thinned out" considerably, and the investigator's path will take him back to "mop up" the remainder.

A simulation was constructed to test this conjecture. A number of points were generated in a square. An investigation was started at a point near the center. The path took the investigator to all the points in the area, and then back to his initial point. Such a closed path is called a tour. Plots of $\ell_k/kE(r_1)$ - vs - k for $k = 1, 2, 3 \ldots n$, $n = 10, 30, 60$ shown in Figure III-5. A cumulative plot of $t_n/nE(r_1)$ - vs - n is shown in Figure III-6.

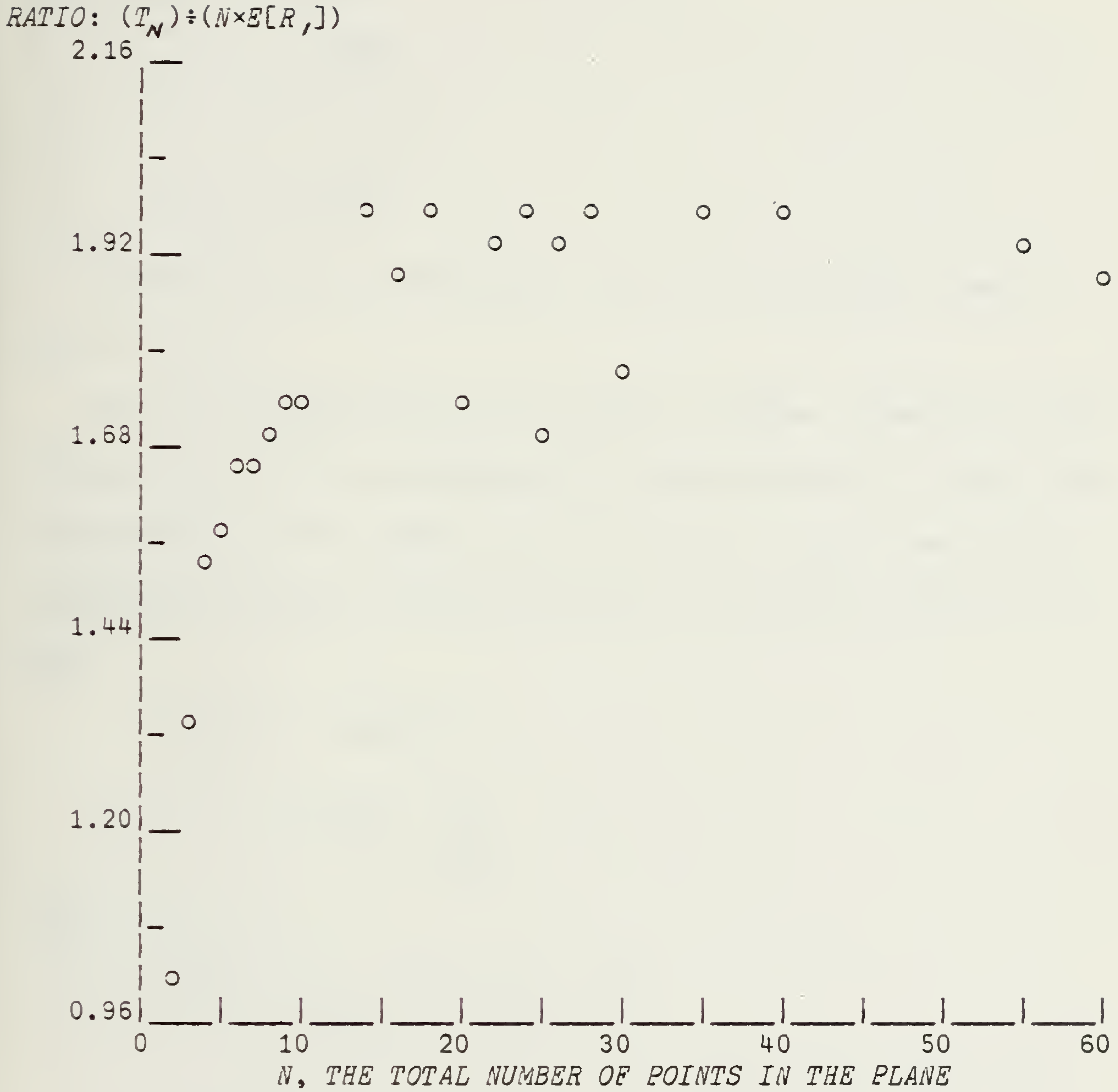
Two things can be seen in these graphs:

1. The "plateau" in the center of the curves in Figure III-5 becomes more pronounced as more points are added to the area. This is a reasonable result.



RATIO OF CUMULATIVE PATH LENGTH, L_K, TO $(K \times E[R_1])$-VS-$K, K=1,2,\ldots,N$

Figure II-5



RATIO OF TOTAL TOUR LENGTH, T_N, *TO* $(N \times E[R_1])$ *=VS= N*

Figure II-6

2. The final cumulative length is approximately

$$t_n = 1.95\frac{(\sqrt{nA})}{2} = 1.95nE(r_1), \; n \geq 7.$$

Plots of repeated simulation trials suggest that t_n is normally distributed with mean

$$\bar{t}_n = 1.95\,\frac{\sqrt{nA}}{2}$$

$$S_{t_n}^{\;2} = 0.06\,\frac{nA}{4.}$$

Stemleaf plots for the cases n = 20, 40, and 60 are shown in Figure III-7.

This is the expression for $\bar{t}_n$ when n is known. Generally n will not be known, but the assumption that the search area is taken from a Poisson field will enable the calculation of $E(t_n)$ from p and A. The expected number of points occurring in an area A is E(n) = pA. Thus

$$E(t_n|n) = 1.95uE(r_1)$$

$$= 1.95\;n\,\frac{1}{2\sqrt{p}}$$

and

$$E(t_n) = \sum_{k=0}^{\infty} 1.95\;k\,\frac{1}{2\sqrt{p}}\,f_n(k)$$

where $f_n(k) = P(n = k)$

$$= 1.95\,\frac{1}{2\sqrt{p}}\,\sum_{k=0}^{\infty} k\;f_n(k)$$

$$= 1.95\,\frac{1}{2\sqrt{p}}\;E(n)$$

$$= 1.95\,\frac{1}{2\sqrt{p}}\cdot pA = 1.95\,\frac{\sqrt{p}\,A}{2}$$

```
    STEMLEAF F20PTS
13|47
14|2567
15|233699
16|00122344568
17|001124589
18|1568899
19|1256
20|12
21|234
22|4
23|1
```

```
    STEMLEAF F40PTS
16|04
17|036
18|1223445567
19|011112222355677779
20|1478
21|001458889
22|29
23|02
```

```
    STEMLEAF F60PTS
16|589
17|245778
18|0122355666668
19|0000344555677888
20|0000112268
21|05
```

Stemleaf Plots $\frac{t_n}{nE(r_1)}$ for n = 20, 40, 60

Figure II-7

Similarly,

$$S_{t_n}^2 \approx 0.06 \frac{pA^{3/2}}{4}$$

III. EXAMPLE

The procedure developed may be applied to the example given in Section II.

Assume that the search area is a square area, 100 miles on a side, with a spatial Poisson parameter of

$$p = 1.1 \times 10^{-3}.$$

The expected length of a tour will be given by $E(t_n) = 1.95 \frac{(\sqrt{nA})}{2}$ thus

$$E(t_n) = 323 \text{ miles.}$$

To check this result, eleven points were selected at random, with one being the searcher. The positions, measured from the southwest corner, were:

Index	x	y
1	32.1	51.1
2	60.5	00.4
3	90.5	17.3
4	49.8	18.2
5	18.4	99.2
6	65.3	72.9
7	40.8	12.8
8	51.6	22.2
9	69.7	99.3
10	58.0	74.0
0	18.0	19.8. (searcher)

These points are plotted in Figure III-1. It is assumed that the searcher's detection range is greater than the greatest inter-target spacing.

The myopic investigation starting at point "0" is shown by the solid line in Figure III-2. I = {0, 7, 4, 8, 2, 3, 6, 10, 9, 5, 1, 0}. The actual length of the myopic investigation starting at pt 0 is 328.66. In this case the myopic path is also the optimal path through all the points, and 328.66 is the minimum length. In this case n = 11 and

$$S_{t_n}^{\ 2} \approx .06 \frac{nA}{4} \quad 1650$$

$$S_{t_n} \approx 40.62$$

So the actual value is well within one standard deviation of the predicted value.

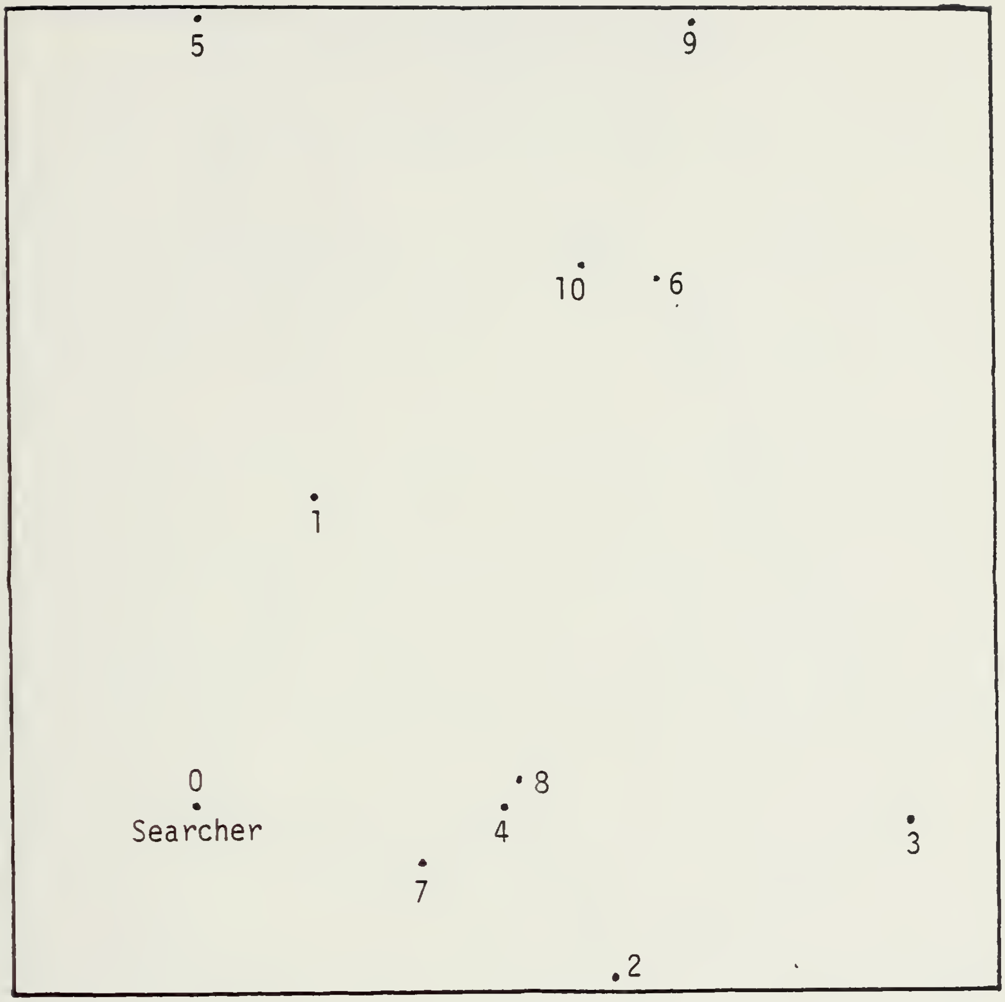


Figure III-1

Point Locations for Example Problem

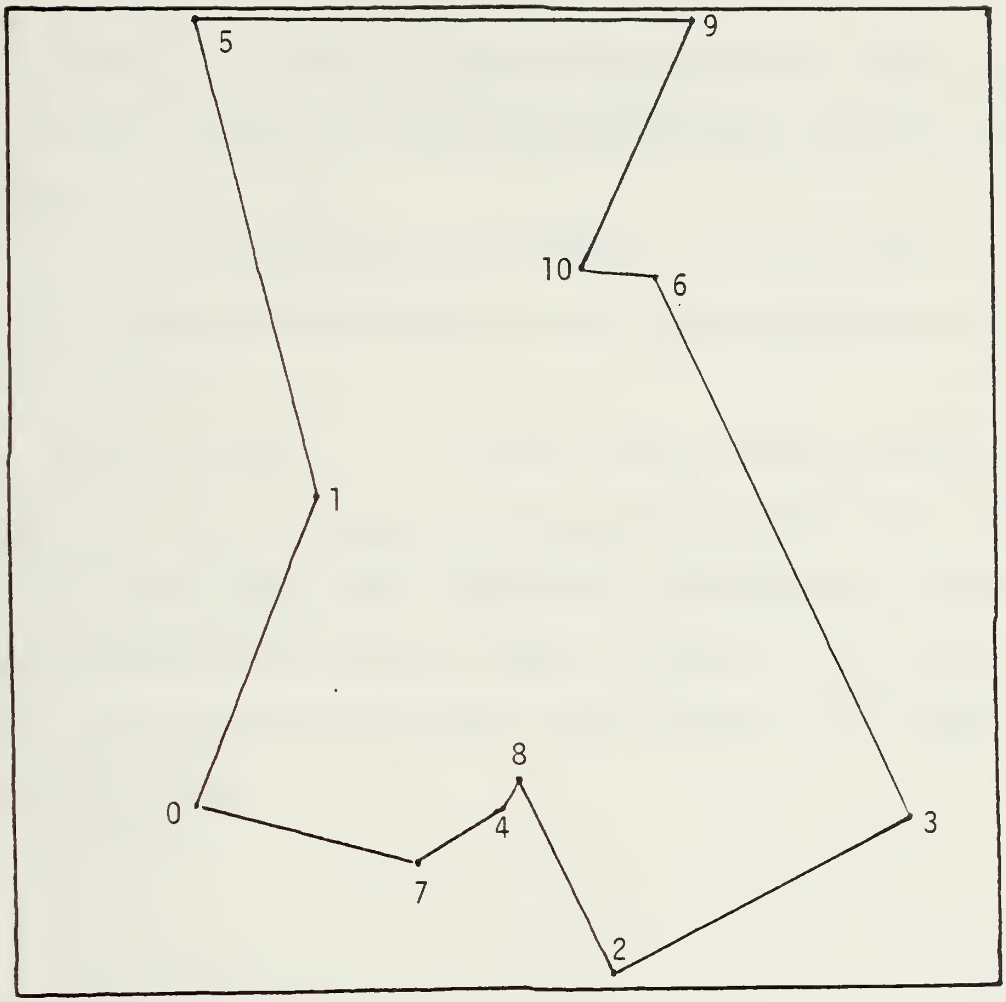


Figure III-2

Myopic Tour Through Example Problem

IV. SUMMARY AND CONCLUSIONS

This thesis has presented a method of estimating the length of a path required to visit all of the targets in a search area under three conditions:

1. The targets are distributed uniformly within the area.

2. The "nearest uninvestigated target" (myopic) decision rule is used.

3. Detection radius is much greater than inter-target spacing.

Simulations results have shown that the expected distance of a complete tour of all points under these assumptions is predicted very well by a simple function of the area and number of targets, if that number is known, or by the area and average target density, if the number of targets is not known.

```
     ∇TRIALS[⎕]∇
    ∇ RATIOS←TRIALS;INDEX;COUNT;A;B;NP
[1]    ⍝ THIS FUNCTION GENERATES 'NTRIALS' SETS OF
[2]    ⍝ POINTS, EACH SET HAVING 'NPOINTS' MEMBERS.  IT CALCULATES
[3]    ⍝ THE EXPECTED LENGTH OF THE FIRST JUMP, AND THEN CALLS 'NUT2'
[4]    ⍝ TO FIND THE LENGTH OF A MYOPIC INVESTIGATION STARTING NEAR THE
[5]    ⍝ CENTER OF EACH POINT SET.  CUMULATIVE AVERAGE RESULTS ARE OUTPUT T
       HROUGH
[6]    ⍝ 'CUMS.  OFFLINE PRINTING OF INTERMEDIATE AND FINAL RESULTS IS CONT
       ROLLED
[7]    ⍝ THROUGH THE VARIABLE QPRINT.
[8]    SIZE←NPOINTS-1
[9]    RATIOS←0
[10]   CMS 'CLOSIO PRINTER OFF'
[11]   '

       '
[12]   'NUMBER OF TRIALS THIS RUN  ';NTRIALS
[13]   'NUMBER OF POINTS GENERATED PER TRIAL  ';NPOINTS
[14]   'SPATIAL POISSON CONSTANT, LAMBDA  ';LAMBDA←NPOINTS÷10*6
[15]   EXLENGTH←÷2×LAMBDA*0.5
[16]   'EXPECTED LENGTH OF FIRST JUMP  ';EXLENGTH
[17]   '
       '
[18]   CUMS←0
[19]   INDEX←NTRIALS
[20]  STEP:NP←NPOINTS
[21]   B←(NPOINTS?1000),[1.5](NPOINTS?1000)
[22]   COUNT←1↑⍴B
[23]  STEP2:A←,B[COUNT;]
[24]   →STEP3×⍳(50≥|500-A[1])×50≥|500-A[2]
[25]   →STEP2×⍳COUNT←COUNT-1
[26]   →STEP
[27]  STEP3:B← 1 0 ↓(COUNT-1)⊖B
[28]   RATIOS←RATIOS,A NUT2 B
[29]   CUMS←CUMS+OUTPUT,[2](OUTPUT[;6]*2),[1.5](OUTPUT[;7]*2)
[30]   →STEP×⍳INDEX←INDEX-1
[31]   CUMS←CUMS÷NTRIALS
[32]   CUMS[;8]←(NTRIALS÷NTRIALS-1)×(CUMS[;8]-CUMS[;6]*2)
[33]   CUMS[;9]←(NTRIALS÷NTRIALS-1)×(CUMS[;9]-CUMS[;7]*2)
[34]   CH←NUT2CH,',L VAR,R VAR'
[35]   ⎕←A←TRIALSFMT FMT CUMS
[36]   →NOFFLINE×⍳QPRINT=0
[37]   OPRINT A
[38]  NOFFLINE:CMS 'CLOSIO PRINTER ON'
[39]   RATIOS←1↓RATIOS
    ∇
```

Computer Program for Simulation

```
      ∇NUT2[⎕]∇
    ∇ PATHLTH←STARTSTOP NUT2 POINTS;DIST;DIST2;UNVISITED;NEXT;NOW;NUM;
      NUM21;START;STOP
[1]   ⍝ THIS FUNCTION FINDS THE MYOPIC INVESTIGATION THROUGH THE
[2]   ⍝ POINT SET 'POINTS' STARTING AT 2↑STARTSTOP AND ENDING AT
[3]   ⍝ ¯2↑STARTSTOP.  OUTPUT IS THROUGH THE ARRAY 'OUTPUT' AND
[4]   ⍝ CONSISTS OF THE LIST OF POINT INDICES (FROM 'POINTS'), POINT
[5]   ⍝ COORDINATES, JUMP LENGTHS, CUMULATIVE LENGTHS, AND RATIOS
[6]   ⍝ OF CUMULATIVE LENGTH TO (N×EXLENGTH), WHERE 'EXLENGTH' IS THE
[7]   ⍝ EXPECTED LENGTH OF THE FIRST JUMP.
[8]   NUM←SIZE
[9]   NUTJUMPS←0
[10]  NUTLENGTHS←0
[11]  LENGTH←0
[12]  NUTPATH←NOW←1
[13]  LIST2←(NUM+1)↑LIST←⍳N←2+1↑⍴POINTS
[14]  START←2↑STARTSTOP
[15]  STOP←¯2↑STARTSTOP
[16]  DIST←DISTANCE START,[1] POINTS,[1] STOP
[17]  DIST←DIST+ZAP×(⍳N)∘.=(⍳N)
[18]  UNVISITED←0,((N-2)⍴1),0
[19] STEP:DIST2←UNVISITED/DIST
[20]  NEXT←1↑⍋,DIST2[NOW;]
[21]  LENGTH←LENGTH+DUMMY←,DIST2[NOW;NEXT]
[22]  NUTJUMPS←NUTJUMPS,DUMMY
[23]  NUTLENGTHS←NUTLENGTHS,LENGTH
[24]  NOW←1↑(NEXT-1)⌽UNVISITED/LIST
[25]  UNVISITED[NOW]←0
[26]  NUTPATH←NUTPATH,NOW
[27]  →STEP×⍳0≠NUM←NUM-1
[28]  NUTPOINTS←START,[1] POINTS[1↓NUTPATH-1;],[1] STOP
[29]  NUTPATH←(NUTPATH-1),0
[30]  NUTLENGTHS←NUTLENGTHS,LENGTH+PATHLTH←,DIST[NOW;N]
[31]  NUTJUMPS←NUTJUMPS,PATHLTH
[32]  OUTPUT←(LIST2,1),[2] NUTPATH,[2] NUTPOINTS,[2] NUTJUMPS,[2]
      NUTLENGTHS,[1.5](NUTLENGTHS÷EXLENGTH×(⍳⍴NUTLENGTHS)-1)
[33]  PATHLTH←OUTPUT[NPOINTS+1;7]
[34]  →OFFLINE×QPRINT≥2
[35] OFFLINE:CH←NUT2CH
[36]  OPRINT NUT2FMT FMT OUTPUT
    ∇
```

Computer Program for Simulation

```
      ∇DISTANCE[⎕]∇
    ∇ DIST←DISTANCE POINTS
[1]   ⍝ THIS FUNCTION GENERATES AN N×N DISTANCE MATRIX
[2]   DIST←(POINTS[;2]∘.-POINTS[;2])*2
[3]   DIST←DIST+(POINTS[;1]∘.-POINTS[;1])*2
[4]   DIST←DIST*0.5
    ∇
```

Computer Program for Simulation

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